u, v = velocities in the boundary layer in x and y directions, respectively
U = velocity of the fluid in the central core
V = average velocity, $Q/2h$
x, y = Cartesian coordinates
x^* = dimensionless axial distance, $x/h\,N_{Re}$
w = mass rate of flow

Greek Letters

δ = boundary-layer thickness
δ^* = dimensionless boundary-layer thickness, δ/h
μ = viscosity
ρ = density
$< >$ = average over cross section available for flow of quantity enclosed
$\Delta( )$ = difference of the quantity enclosed at an arbitrary cross section and at the entry

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Investigation of Heat Transfer to Liquid Metals Flowing in Circular Tubes

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In a recent article, Dwyer (*1*) has discussed the heating and cooling of turbulent flowing liquids in circular tubes. A result of this analysis is that the equation

$$N_{Nu} = 7.0 + 0.025 \left[N_{Pe} - \frac{1.82\,N_{Re}}{(\epsilon_M/\nu)^{1.4}_{\max}} \right]^{0.8} \quad (1)$$

could be utilized to estimate heat transfer rates to liquid metals flowing in circular tubes under fully established flow and constant heat flux conditions. Equation (1) applies for Peclet numbers above the critical Peclet number. Below the critical Peclet number it is recommended that the Nusselt number be assumed equal to 7.0. The critical Peclet number is defined as that Peclet number corresponding to the highest Reynolds number, for a given Prandtl number, at which heat is transferred by molecular conduction only.

The results of Lyon (*2*) in an earlier study indicate that the semi-empirical relationship

$$N_{Nu} = 7.0 + 0.025\,(N_{Pe})^{0.8} \quad (2)$$

may be utilized to estimate heat transfer coefficients under turbulent flow and uniform heat input conditions for liquid metals flowing in tubes.

Equations (1) and (2) are in fair agreement with each other. Recent data from the Brookhaven National Laboratory (*3*) appear to be in reasonable agreement with both expressions.

Other investigators have reported lower results. Lubarsky and Kaufman (*4*) have presented the expression

$$N_{Nu} = 0.625\,N_{Pe}^{0.4} \quad (3)$$

for fully developed turbulent heat transfer to liquid metals flowing in tubes. Reference 5 discusses the various effects which contribute to the disagreement of experimental measured liquid-metal heat transfer coefficients.

In the laminar flow region, the Nusselt number should equal 4.4 (*1*). This is in agreement with the experimental results of Petukhov and Yushin (*6*).

The purpose of this communication is to report information gained in an experiment in which a constant heat flux was supplied to liquid sodium-potassium flowing vertically in a circular tube. Data obtained from this experiment are in the laminar, transition, and turbulent regions.

EXPERIMENTAL APPARATUS

A schematic diagram of the experimental loop is shown in Figure 1. The loop was constructed primarily of 0.500 in. O.D., 0.035 in. wall, type 316 stainless steel and was contained within an evacuated enclosure. Sodium-potassium alloy (NaK-78) was used as the heat transfer fluid. The loop contained a 16-in. long thermal radiation heated test section. Radiation shields surrounded the heated section to reduce heat losses. Heat was removed from the loop by means of a cooling coil designed to remove heat by thermal radiation. An electromagnetic pump was used for circulating the sodium-potassium and an e.m. flowmeter was used for obtaining flow rates. Chromel-alumel thermocouples were employed to measure temperatures at key points in the system. The loop was pressurized by utilizing an argon gas blanket on the sodium-potassium surface in the dump tank.

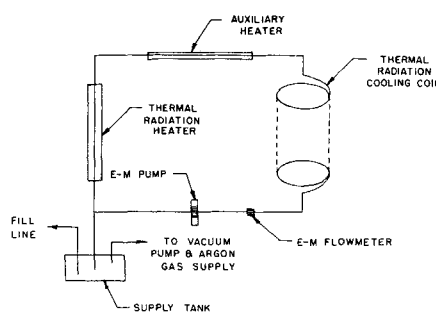

Fig. 1. Schematic diagram of thermal radiation heated loop.

The vertical thermal radiation heated test section consisted of a 0.500-in. O.D., 0.035-in. wall, type 316 stainless steel tube containing the flowing sodium-potassium. The tube was surrounded by two half cylindrical shaped tantalum elements which were 16 in. long and electrically heated. The test section had an L/D ratio of 37. Heat was transferred from the 0.003-in. thick heating elements to the tube containing the flowing sodium-potassium by thermal radiation. The heat was then conducted through the tube wall to the liquid sodium-potassium.

The heat flux was determined both by utilizing an energy balance on the sodium-potassium passing through the thermal radiation heater and from power measurements less calculated losses. These two techniques for determining the heat flux showed excellent agreement (within ± 4%). The heat flux was uniform along the heated length except for a sharp decrease within approximately 1 in. of both the heater inlet and heater outlet.

Two chromel-alumel thermocouples were located both at the inlet and at the outlet of the heated section to measure the sodium-potassium temperature at these points. These thermocouples were located several inches from both the actual heater inlet and heater outlet to eliminate any heater end effects upon the thermocouple readings and to assure proper mixing of the fluid at the heater outlet. With the inlet and outlet temperatures and heat flux known, the bulk fluid temperature was determined as a function of length along the vertical heater. Three chromel-alumel thermocouples were spot welded to the outside tube wall in the middle of

INFORMATION RETRIEVAL

(*Continued from page 1150*)

Direct contact heat transfer with change of phase: Effect of the initial drop size in three-phase heat exchangers, Sideman, Samuel, Gideon Hirsch, and Yehuda Gat, **A.I.Ch.E. Journal, 11,** No. 6, p. 1081 (November, 1965).

Key Words: A. Drops-6, 9, Bubbles-6, 9, Diameter-6, Heat Transfer-7, 8, Heat Transfer Coefficient-7, 8, Heat Exchangers-9, Three-Phase-0, Coalescence-6, Turbulence-6.

Abstract: Initial diameters of drops of volatile fluids evaporating in immiscible liquids were found to affect the heat transfer coefficients in single- and multidrop systems. The effect in the latter systems decreases with increased coalescence.

The influence of gravitational force on gas absorption in a packed column, Vivian, J. E., P. L. T. Brian, and V. J. Krukonis, **A.I.Ch.E. Journal, 11,** No. 6, p. 1088 (November, 1965).

Key Words: Gravity-6, Gas Absorption-7, 8, Packed Column-10, Carbon Dioxide-2, Water-5, Centrifuge-10, Centrifugal Force-6, Mass Transfer-7.

Abstract: Carbon dioxide was desorbed from water in a packed column mounted on a large centrifuge. By varying the centrifuge rotational speed, the total body force acting upon the liquid was varied from 1 to 6.4 times the normal gravitational force. The results, obtained for ¾ in. Raschig ring packing, showed K_LA varying with the body force **G** to a power between 0.41 and 0.48, the exponent being larger at lower liquid flow rated. This effect of **G** is compared with penetration theory predictions and with that inferred from dimensional analysis and the effect of packing size.

Multicomponent vapor-liquid equilibria at high pressures: Part I. Experimental study of the nitrogen–oxygen–carbon dioxide system at 0°C., Muirbrook, N. K., and J. M. Prausnitz, **A.I.Ch.E. Journal, 11,** No. 6, p. 1092 (November, 1965).

Key Words: Equilibrium-8, Phase Equilibrium-8, Vapor-Liquid Equilibrium-8, Molar Volume-7, 8, Critical Properties-8, Critical Pressure-8, Critical Composition-8, Ternary Systems-9, Binary Systems-9, Nitrogen-9, Carbon Dioxide-9, Oxides (Inorganic)-9, Oxygen-9, Condensation-6, Pressure-6, Composition-6, Volume-7, Cells-10, Pumps-10, Vane Pumps-10, Balances-10, Extrapolation-10.

Abstract: A description is given of a dynamic vapor-liquid equilibrium apparatus in which both phases are recirculated by vane pumps especially designed for high-pressure service. Equilibrium phase compositions as well as vapor and liquid molar volumes up to the critical pressure are reported for the two binary systems nitrogen-carbon dioxide and oxygen-carbon dioxide and for the ternary.

Multicomponent vapor-liquid equilibria at high pressures: Part II. Thermodynamic analysis, Chueh, P. L., N. K. Muirbrook, and J. M. Prausnitz, **A.I.Ch.E. Journal, 11,** No. 6, p. 1097 (November, 1965).

Key Words: Equilibrium-8, Thermodynamics-8, Fugacity-7, 8, Activity Coefficients-7, 8, Vapor-Liquid Equilibrium-8, **K** Factors-8, Molar Volume-8, Volume-8, Multicomponent Systems, 9, Binary Systems-9, Liquids-9, Vapors-9, Carbon Dioxide-9, Oxides (Inorganic)-9, Oxygen-9, Nitrogen-9, Temperature-6, Pressure-6, Composition-6, Models-10, Equations-10, Van Laar Model-10, Separation-4, Absorption-4.

Abstract: Thermodynamic analysis is applied to high-pressure vapor-liquid equilibria in binary and multicomponent mixtures containing one or more noncondensable components. A thermodynamic consistency test is described, and a modification of van Laar's model is given for representing adjusted activity coefficients in binary and multicomponent liquid solutions up to the critical composition. With only experimental data used on the two binaries at 0°C., vapor-liquid equilibria are calculated for the carbon dioxide–nitrogen–oxygen ternary at high pressures. Brief reference is made to the possibility of air separation by high-pressure absorption in liquid carbon dioxide.

(*Continued on page 1154*)

the heated length. With this outside wall temperature and heat flux known, the inside wall temperature was calculated from the expression

$$a = \frac{2\pi KL\,(T_o - T_i)}{\ln\,(D_o/D_i)} \qquad (4)$$

The system was thoroughly cleaned prior to assembly, and during the filling of the system several techniques were used to remove oxides from the sodium-potassium. Also, between periods of loop operation sodium-potassium was dumped from the loop into the dump tank. During these dumps the heavier oxides settled to the dump tank bottom and the lighter oxides rose to the top of the sodium-potassium level. Thus, it was assumed that very little oxide was contained in the sodium-potassium which was being circulated in the loop.

RESULTS AND DISCUSSION

The thermal radiation heated test section was operated for approximately 700 hr. During this time sodium-potassium temperatures up to 1,300°F. heat fluxes up to 62,000 B.t.u./(hr.)(sq. ft.), and flow rates up to 416 $lb._m$/hr. were obtained. Data were obtained in the laminar, transition, and turbulent flow regions.

Figures 2 and 3 show the data from this experiment and the results of Dwyer (*1*) for estimating heat transfer to liquid metals flowing in circular tubes. Figure 2 shows the Nusselt number vs. Reynolds number plot and Figure 3 shows the plot of Nusselt number vs. Peclet number. The data points obtained from this experiment are in fair agreement with the recommended prediction of Dwyer.

An L/D ratio of 20 existed before the entrance into the heated section. Sleicher and Tribus (7) have examined entry lengths for fluids with low Prandtl numbers. These results indicate that for low Reynolds numbers (less than around 2.5×10^4) an entrance length of ten diameters or less is sufficient for establishing fully developed turbulent flow. Also, due to the high thermal conductivity of sodium-potassium, the velocity profile

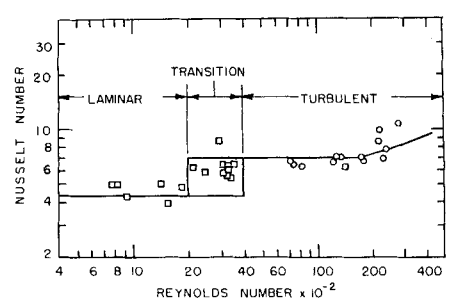

Fig. 2. Plot of Nusselt number vs. Reynolds number.

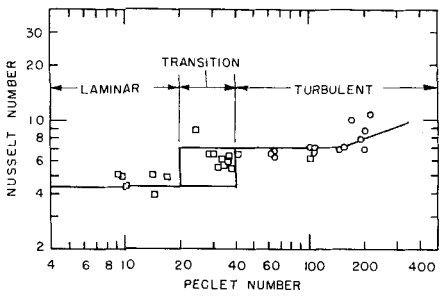

Fig. 3. Plot of Nusselt number vs. Reynolds number.

should have a minimal effect upon the the heat transfer coefficient.

The accuracy of the data obtained from this experiment was checked by means of an error analysis. The experimental uncertainties in the values for Nusselt number were found to range from ± 6 to ± 16% for the square data points in Figures 2 and 3, and uncertainties ranging from ± 10 to ± 22% were found for the round data points. The uncertainty in the Reynolds numbers was ± 5% and the uncertainty in the Prandtl numbers was ± 2%. The possible errors in the data were determined by using the statistical method. This method consists of taking the square root of the sum of the squares of the individual experimental uncertainties to determine the possible value of error.

The accuracy of the Nusselt number calculated from experimental data is largely dependent upon accurate determination of the difference between the inside wall temperature and the bulk fluid temperature. In the determination of the possible error in calculating the Nusselt number, we considered the following possible sources of error: heat flux, thermal conductivity of SS tubing, heater inlet temperature, heater outlet temperature, and outside wall temperature. In determining the accuracy of the Reynolds numbers, we considered the accuracies with which the flow rate could be experimentally determined and of the published thermophysical property data (*8*). The accuracy of the thermophysical property data was also considered in determination of the Prandtl numbers.

The data points shown in Figures 2 and 3 represent an average of three to eight individual data points at which the flow rates and bulk fluid temperatures were identical; thus the Reynolds numbers and Prandtl numbers were identical. The accuracy of these individual data points was within the experimental accuracy limits determined by the error analysis.

Some of the earlier results which showed the Nusselt numbers to be somewhat lower than predicted by Equations (1) and (2) may be explained by nonwetting of the heat transfer surface by the liquid metal. Recent data (*3, 9*) have extended the temperature range of these experiments with the alkali metals, thus assuring complete wetting of the heat transfer surface. These data (*3, 9*) are in agreement with Equations (1) and (2).

The data from this experiment support the equations presented by Lyon (*2*) and Dwyer (*1*) for estimating heat transfer rates to liquid metals during vertical upflow.

ACKNOWLEDGMENT

The author wishes to thank Dr. Paul A. Lottes and Joel B. Heineman for helpful discussions and suggestions during this study. He also acknowledges the assistance of Angelo Mele in building and operating the experimental apparatus.

NOTATION

D_o = O.D. of heated tube, ft.
D_i = I.D. of heated tube, ft.
K = thermal conductivity of tube, B.t.u./(hr.)(ft.)(°F.)
L = heated length, ft.
N_{Nu} = Nusselt number, dimensionless
N_{Pe} = Peclet number, dimensionless
N_{Pr} = Prandtl number, dimensionless
q = heat conducted through the tube, B.t.u./hr.
N_{Re} = Reynolds number, dimensionless
T_o = local outside tube wall temperature, °F.
T_i = local inside tube wall temperature, °F.

Greek Letters

ϵ_M = eddy diffusivity for momentum transfer, sq.ft./hr.
ν = kinematic viscosity, sq.ft./hr.

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A Criterion for Short and Long Wetted-Wall Columns

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A simple and convenient length criterion for practical purposes is whether the Fourier number is more or less than 0.1.

Consider the case of fully developed laminar flow of a liquid B down the walls of a cylinder with a radius sufficiently large so that the effects of curvature are negligible. Impose on this system mass transfer of component A from a gas such that the concentration of A at the liquid-gas interface is constant with length. Mass transfer in this liquid phase has been shown by several authors to be

$$U_s\left[1-\left(\frac{x}{h}\right)^s\right]\frac{\partial C}{\partial z}=D_{AB}\frac{\partial^2 C}{\partial x^2} \quad (1)$$

$$C=C_o \quad \text{at} \quad z=0 \quad (2)$$

$$C=C_s \quad \text{at} \quad x=0 \quad (3)$$

$$\partial C/\partial x=0 \quad \text{at} \quad x=h \quad (4)$$

These equations are written with the coordinate origin at the top of the liquid-gas interface.

Equation (1) is precise only where the molecular diffusion of A in the z direction is negligible compared to transport of A by fluid motion in the z direction, and where conditions are such that the difference between the flux of A in Lagrangian and Eulerian forms is negligible.

Equation (1) has been solved by Pigford (3), who used boundary con-